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ON SOLVING NONSTANDARD EQUATIONS BY ALGEBRAIC METHODS

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ABSTRACT

The article presents methods for solving nonstandard equations using algebraic transformations and substitutions. Several examples are analyzed to demonstrate effective approaches that develop students' mathematical thinking and problem-solving abilities.

Keywords: Nonstandard equations, algebraic methods, auxiliary substitution, trigonometric substitution, problem solving, mathematical thinking, creative thinking, mathematics education, elementary functions, analytical skills.

INTRODUCTION

A nonstandard problem is a problem for which there is no fixed algorithm or generally applicable rule for solution. A problem that is considered standard in one course of mathematics may become nonstandard in another course. Solving nonstandard problems is an important form of educational activity for students, because in this process mathematical thinking and creative ability develop, which, in turn, contributes to improving the effectiveness of mathematics education.

The ability to solve nonstandard problems is acquired through practice. At present, the rapid development of socio-economic life requires academic lyceum students to possess comprehensive and solid knowledge. In particular, learning to solve nonstandard problems helps students develop nonstandard thinking and creative reasoning skills. Below we present several examples of solving nonstandard equations by algebraic methods, especially by introducing auxiliary substitutions.

Equation 1. Solve the equation

$$x^3 + 1 = 2\sqrt[3]{2x - 1}.$$

Solution. First, introduce the substitution

$$\sqrt[3]{2x - 1} = a.$$

Then

$$2x - 1 = a^3, \qquad 2x = a^3 + 1.$$

Taking the introduced notation into account in the given equation, we obtain

$$x^3 + 1 = 2a.$$

At the same time, from the substitution we have

$$a^3 + 1 = 2x.$$

Subtracting the latter relation from the former in a suitable form gives

$$x^3 - a^3 = -2(x - a).$$

Hence,

$$(x - a)(x^2 + ax + a^2) + 2(x - a) = 0,$$

or

$$(x - a)(x^2 + ax + a^2 + 2) = 0.$$

The second factor is always positive for real values of x and a, because

$$x^2 + ax + a^2 + 2 = \left(x + \frac{a}{2}\right)^2 + \frac{3a^2}{4} + 2 > 0.$$

Therefore,

$$x - a = 0, \qquad x = a.$$

Thus,

$$\sqrt[3]{2x - 1} = x.$$

Cubing both sides yields

$$x^3 - 2x + 1 = 0.$$

The roots of this equation are

$$x \in \left\{1, \ \frac{\sqrt{5} - 1}{2}, \ -\frac{\sqrt{5} + 1}{2}\right\}.$$

Therefore, the solution set of the original equation is

$$\boxed{\left\{1, \ \frac{\sqrt{5} - 1}{2}, \ -\frac{\sqrt{5} + 1}{2}\right\}}.$$

Equation 2. Solve the equation

$$x + \frac{x}{\sqrt{x^2 - 1}} = \frac{35}{12}.$$

Solution. Since the right-hand side is positive and the expression is defined for $|x| > 1$, we consider $x > 1$. Introduce the trigonometric substitution

$$x = \frac{1}{\sin t}, \qquad 0 < t < \frac{\pi}{2}.$$

Then

$$\sqrt{x^2 - 1} = \sqrt{\frac{1}{\sin^2 t} - 1} = \frac{\cos t}{\sin t},$$

and consequently

$$\frac{x}{\sqrt{x^2 - 1}} = \frac{1}{\cos t}.$$

Thus, the equation becomes

$$\frac{1}{\sin t} + \frac{1}{\cos t} = \frac{35}{12},$$

or

$$\frac{\sin t + \cos t}{\sin t \cos t} = \frac{35}{12}.$$

Squaring both sides, we obtain

$$\frac{\sin^2 t + 2\sin t\cos t + \cos^2 t}{\sin^2 t\cos^2 t} = \frac{35^2}{12^2}.$$

Put

$$\sin t\cos t = a.$$

Then

$$\frac{1 + 2a}{a^2} = \frac{35^2}{12^2}.$$

Solving this quadratic equation with respect to a gives

$$a_1 = \frac{12}{25}, \qquad a_2 = -\frac{12}{49}.$$

Since $0 < t < \frac{\pi}{2}$, we have $a = \sin t\cos t > 0$, so only

$$a = \frac{12}{25}$$

is acceptable. Hence,

$$\sin 2t = 2\sin t\cos t = \frac{24}{25}.$$

Using the identity $\sin^2 2t + \cos^2 2t = 1$, we find

$$\cos 2t = \frac{7}{25}.$$

Therefore,

$$\sin t = \frac{3}{5} \quad \text{or} \quad \sin t = \frac{4}{5}.$$

Returning to $x = \frac{1}{\sin t}$, we obtain

$$x = \frac{5}{3} \quad \text{or} \quad x = \frac{5}{4}.$$

Thus, the equation has the roots

$$\boxed{x \in \left\{\frac{5}{3}, \frac{5}{4}\right\}}.$$

Equation 3. Solve the equation

$$32 + x = \sqrt[5]{2 - x}.$$

Solution. Introduce the substitution

$$a = \sqrt[5]{2 - x}.$$

Then

$$2 - x = a^5$$

and, since $32 = 2^5$,

$$2^5 + x = a.$$

Adding the two obtained relations, we get

$$a^5 + a = 2^5 + 2.$$

That is,

$$a^5 + a = 34.$$

It is easy to notice that $a = 2$ is a root of this equation. Moreover, the function

$$f(a) = a^5 + a$$

is strictly increasing on the real line, because

$$f'(a) = 5a^4 + 1 > 0.$$

Therefore, $a = 2$ is the unique real root. Substituting $a = 2$ into

$$32 + x = a,$$

we find

$$32 + x = 2, \qquad x = -30.$$

Thus, the given equation has the unique root

$$\boxed{x = -30}.$$

The same conclusion can also be obtained by factoring:

$$a^5 - 2^5 + a - 2 = 0,$$

$$a^5 - 2^5 + a - 2 = (a - 2)(a^4 + 2a^3 + 4a^2 + 8a + 17) = 0.$$

This again gives $a = 2$, hence $x = -30$.

Equation 4. Solve the equation

$$\sin(x^3 + 2x^2 + 1) = x^2 + 2x + 3.$$

Solution. Rewrite the right-hand side by completing the square:

$$x^2 + 2x + 3 = (x + 1)^2 + 2.$$

Since

$$-1 \le \sin(x^3 + 2x^2 + 1) \le 1,$$

while

$$(x + 1)^2 + 2 \ge 2,$$

the left-hand side belongs to the interval $[-1,1]$, whereas the right-hand side belongs to $[2, \infty)$. These two intervals do not intersect. Therefore, the equation has no real solution:

$$\boxed{\varnothing}.$$

Equation 5. Solve the equation

$$\sqrt{x^2+1} - x = \frac{5}{2\sqrt{x^2+1}}.$$

Solution. To solve this equation, introduce the substitution

$$x = \tan t.$$

Then

$$\sqrt{x^2+1} = \sqrt{\tan^2 t + 1} = \frac{1}{\cos t}.$$

The equation takes the form

$$\frac{1}{\cos t} - \tan t = \frac{5\cos t}{2}.$$

Equivalently,

$$\frac{1-\sin t}{\cos t} = \frac{5\cos t}{2}, \qquad \cos t \neq 0.$$

Multiplying both sides by $2\cos t$, we obtain

$$2(1-\sin t) = 5\cos^2 t.$$

Using $\cos^2 t = 1 - \sin^2 t$, we get

$$2(1-\sin t) = 5(1-\sin^2 t).$$

Thus,

$$2(1-\sin t) = 5(1-\sin t)(1+\sin t).$$

Since $\sin t = 1$ does not satisfy the original equation, we divide by $1 - \sin t$ and obtain

$$2 = 5(1+\sin t).$$

Therefore,

$$\sin t = -\frac{3}{5}.$$

Taking $\cos t = \frac{4}{5}$, we get

$$x = \tan t = \frac{\sin t}{\cos t} = -\frac{3}{4}.$$

Hence, the equation has the root

$$\boxed{x = -\frac{3}{4}}.$$

In conclusion, solving nonstandard problems by introducing appropriate substitutions and by using the properties of elementary functions leads to effective results in overcoming problematic situations encountered by students. Solving a mathematical problem is an exercise that develops thinking. In addition, problem solving helps develop patience, perseverance and willpower, and gives students a deep sense of satisfaction associated with finding a successful solution.

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